Lesson 13-3
Converse, Inverse, and Contrapositive

Learning Targets:
- Write and determine the truth value of the converse, inverse, and contrapositive of a conditional statement.
- Write and interpret biconditional statements.

Every conditional statement has three related conditionals. These are the converse, the inverse, and the contrapositive of the conditional statement.

When you **interchange** a hypothesis and a conclusion, you switch them. When you **negate** a hypothesis or a conclusion, you rewrite it by adding the word _not_. Note that if a hypothesis or a conclusion already includes the word _not_, you can negate it by removing _not_.

### ACADEMIC VOCABULARY

- **Conditional**: If \( p \), then \( q \).
- **Converse**: If \( q \), then \( p \).
- **Inverse**: If not \( p \), then not \( q \).
- **Contrapositive**: If not \( q \), then not \( p \).

#### 1. Given the conditional statement:
If a figure is a triangle, then it is a polygon.

Complete the table.

<table>
<thead>
<tr>
<th>Form of the statement</th>
<th>Write the statement</th>
<th>True or False?</th>
<th>If the statement is false, give a counterexample.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional statement</td>
<td>If a figure is a triangle, then it is a polygon.</td>
<td>True</td>
<td></td>
</tr>
<tr>
<td>Converse of the conditional statement</td>
<td>If a figure is a polygon, then it is a triangle.</td>
<td>False</td>
<td>Sample answer: quadrilateral</td>
</tr>
<tr>
<td>Inverse of the conditional statement</td>
<td>If a figure is not a triangle, then it is not a polygon.</td>
<td>False</td>
<td>Sample answer: quadrilateral</td>
</tr>
<tr>
<td>Contrapositive of the conditional statement</td>
<td>If a figure is not a polygon, then it is not a triangle.</td>
<td>True</td>
<td></td>
</tr>
</tbody>
</table>

### Universal Access

- Emphasize that to write the statement in the table, students need only be concerned with the structure of the statement. Some students find it helpful to write the condition in one color and the conclusion in another color. Then the colors are reversed for the converse. To negate either the condition or the conclusion, students can write the word _NOT_ in capital letters but keep the colors the same as in the original statement.

### Developing Math Language

As needed, pronounce new terms clearly and monitor students' pronunciation of terms in their class discussions. Use the class Word Wall to keep new terms in front of students. Include pronunciation guides as needed. Encourage students to review the Word Wall regularly and to monitor their own understanding and use of new terms in their group discussions.
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If a given conditional statement is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are logically equivalent.

2. Write a true conditional statement whose inverse is false.
   Sample answer: If an angle has a measure of 95°, then it is obtuse.

3. Write a true conditional statement that is logically equivalent to its converse.
   Sample answer: If today is Monday, then tomorrow is Tuesday.

When a statement and its converse are both true, they can be combined into one statement using the words "if and only if." An "if and only if" statement is a biconditional statement. All definitions you have learned can be written as biconditional statements.

4. Write the definition of perpendicular lines in biconditional form.
   Two lines are perpendicular if and only if they intersect to form right angles.

5. Consider the statement: Numbers that do not end in 2 are not even.
   a. Rewrite the statement in if-then form and state whether it is true or false.
      If a number does not end in 2, then it is not even. False.

   b. Write the converse and state whether it is true or false. If false, give a counterexample.
      If a number is not even, then it does not end in 2. True.

   c. Write the inverse and state whether it is true or false.
      If a number ends in 2, then it is even. True.

   d. Write the contrapositive and state whether it is true or false. If false, give a counterexample.
      If a number is even, then it ends in 2. False. Sample counterexample: 10.

   e. Can you write a biconditional statement for the original statement? Why or why not?
      No. The if-then form of the statement and the converse are not both true.

MATH TERMS
The truth value of a statement is the truth or falsity of that statement.

MATH TIP
Given the biconditional statement "p if and only if q," then the following conditional statements are true.
- If p, then q.
- If q, then p.
- If not p, then not q.
- If not q, then not p.

Activity 13
• The Axiomatic System of Geometry 241

Differentiating Instruction
Support students who are struggling with these concepts to write several if-then statements and to label the condition and the conclusion. Beneath each statement have them write the converse and determine the truth value of each. They should see that a true conditional statement may or may not have a converse that is true and vice versa.

2–5 Interactive Word Wall, Vocabulary Organizer, Jigsaw
As students work in small groups, have one student in each group become the "expert" in writing the converse of a conditional statement, another the inverse, and another the contrapositive. Each "expert" should be prepared with an example that is true and another that is false, if possible. Have "experts" then share with the other students in their groups.
ACTIVITY 13
Check Your Understanding
Debrief this lesson to ensure that students understand how to form conditional statements.

Answers
6. If an angle is a right angle, then it measures 90°. If an angle measures 90°, then it is a right angle. If an angle is not a right angle, then it does not measure 90°. If an angle does not measure 90°, then it is not a right angle.
7. The converse is false, and the contrapositive is true.

ASSESS
Students’ answers to the Lesson Practice items will provide a formative assessment of their understanding of determining the truth value of the converse, inverse, and contrapositive of conditional statements and writing and interpreting biconditional statements, and of students’ ability to apply their learning.

Short-cycle formative assessment items for Lesson 13-3 are also available in the Assessment section on SpringBoard Digital.

Refer back to the graphic organizer the class created when unpacking Embedded Assessment 1. Ask students to use the graphic organizer to identify the concepts or skills they learned in this lesson.

ADAPT
Check students’ answers to the Lesson Practice to ensure that they understand the basics of deductive reasoning. Watch for students who confuse a conditional statement with its converse. Have students who need more practice work with nonmathematical if-then statements. For example, students can write the converse of a statement such as If it is rainy, then clouds are in the sky. Provide several such statements and have students highlight the “p” and the “q” parts of the statement, and then switch these to form the converse. A table like the one in Item 1 may be helpful in evaluating the truth values.

See the Activity Practice on page 243 and the Additional Unit Practice in the Teacher Resources on SpringBoard Digital for additional problems for this lesson. You may wish to use the Teacher Assessment Builder on SpringBoard Digital to create custom assessments or additional practice.

8. No. The statements do not have the same truth value. The first statement is true, but the second statement is false.
9. No. Sample explanation: The converse is “If a number is divisible by 2, then it is divisible by 6.” A counterexample shows that the converse is false. The number 8 is divisible by 2, but it is not divisible by 6.

10. a. If an animal is a bird, then it has wings.
   b. No. Sample explanation: The converse is “If an animal has wings, then it is a bird.” A counterexample shows that the converse is false. A butterfly has wings, but it is not a bird. The converse is false, so you cannot write the original statement as a biconditional statement.

LESSON 13-3 PRACTICE
Use the following statement for Items 11–13.
If a vehicle has four wheels, then it is a car.
11. Write the converse.
12. Write the inverse.
13. Write the contrapositive.
14. Write the definition of the vertex of an angle as a biconditional statement.
15. Give an example of a true statement that has a false converse.
   Given: (1) If X is blue, then Y is gold.
   (2) Y is not gold.
   Which of the following must be true?
   A. Y is blue.
   B. Y is not blue.
   C. X is not blue.
   D. X is gold.